The influence of surface slope on the shape of river basins: Comparison between nature and numerical landscape simulations

Sébastien Castelltorta a,b,*, Philippe Yamato a,c

a Earth Surface Dynamics, Geological Institute, Department of Earth Sciences, ETH Zürich, Sonneggstrasse 5, 8092 Zürich, Switzerland
b Section of Earth and Environmental Sciences, Rue des Maralchers 14, 1205 Geneva, Switzerland
c Géosciences Rennes, UMR 6118, Université de Rennes 1, CNRS, Rennes, France

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A B S T R A C T

We investigate the influence of initial conditions of slope and surface roughness on the shape (length to width aspect ratio) of incipient drainage basins in numerical experiments of simple tilted surfaces using the CASCADE code of landscape evolution. Comparison with data on the shape of river basins in nature shows that simple rules of the steepest-descent routing of water are sufficient to account for a natural range of incipient drainage basin shape, independently of the erosion processes at work. To produce numerical basins that respect the main aspect ratio of natural drainage basins, one must use very low initial regional surface slopes of less than 1° at the scale of the entire drainage basins, and a local roughness slope of less than 3° at the scale of local surface irregularities. Numerical studies addressing real study cases may take advantage of the relation between local roughness and regional slope in order to produce catchment aspect ratios similar to the natural studied cases.

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1. Introduction

In the last three decades, many questions have been addressed about the possible couplings between deep (mantle) and shallow (crustal) tectonics and superficial climate-controlled erosion (e.g., Molnar and England, 1990; Willett, 1999). These led to a considerable improvement in the mathematical modeling of surface processes, in particular with the advent of numerical models of landscape evolution, and with the quantitative analysis of increasingly available high-resolution datasets of the Earth’s topography. While the approaches on these topics have followed different paths, the main studies have focused on numerical modeling of the development and evolution of fluvial landscapes, i.e. landscapes composed of channels and hillslopes, because their structure constitutes a fundamental control on relief in tectonically active areas.

While numerical models of erosion usually produce fluvial landscapes that look similar to nature, how to objectively assess this similarity remains an outstanding problem. As summarized in the review by Tucker and Hancock (2010), questions like what are the essential characteristics of a catchment or a landscape, and how can we quantitatively assess landscape differences (Hancock, 2003) are still challenging. Until now, models have been tested against data obtained with the slope–area relationship or with catchment hypsometry (Hancock et al., 2002; Willgoose et al., 2003) but these measures are often insufficient to discriminate between landscapes that seem visually different (Tucker and Hancock, 2010). Braun and Sambridge (1997) provided a quantitative assessment of the similarity between their model’s results and natural landscapes by showing that the numerically produced dendritic river patterns respected the main laws of network composition and topographic surfaces that have similar scaling behavior as natural landscapes. They noted, however, that most possible networks, either natural or not, inevitably obey Horton’s and Schumm’s laws of network composition, as pointed out by Kirchner (1993), and thus these laws may not be used readily to validate models.

One consistent output from model-data comparison tests is the sensitivity of models to initial conditions. Testing the SIBERIA landscape evolution model (Willgoose et al., 1991a,b,c) against common geomorphological statistics (e.g., Horton’s and Tokunaga’s ratios, non-dimensional drainage density, magnitude, mean relief, and mean stream relief), Ijjasz-Vasquez et al (1992) concluded that the large variability observed in their numerical dendritic networks is not random and is instead directly related to differences in the initial conditions. A subsequent test of the SIBERIA model by Hancock (2003) also concluded that, for a good match between simulated and field descriptors such as the hypsometric curve, area–slope relationship, width function and cumulative area, all that is needed is a catchment with an aspect ratio matching that of the field data, thus similarly emphasizing the role of initial conditions.

Recently, Castelltort et al. (2009) presented new measures of the shape (length to width aspect ratio) of large-scale (101–103 km2)
incipient drainage basins formed on uniformly tilted surfaces. These
data highlight the influence of regional surface slope versus surface
roughness on the aspect ratio of incipient drainage basins. Steep
and smooth slopes develop longer and narrower catchments than
comparatively gently dipping and rougher surfaces.

In this brief report we use Castelltort et al.'s (2009) dataset to test
the ability of the CASCADE landscape evolution model (Braun and
Sambridge, 1997) to accurately reproduce drainage basin shape. The
algorithm of routing water downstream used in the CASCADE is ge-
eric in essence (steepest slope) and is sufficient to reproduce the
observations of natural basin shape. By tuning the ratio between re-
gional slope and amplitude of the initial surface roughness, the initial
drainage basin shape can be predicted. Numerical landscape evolu-
tion studies can thus choose the adequate initial conditions in order
to generate drainage basins whose aspect ratios respect those com-
monly encountered in nature.

2. Incipient river basins on tilted surfaces

To measure the aspect ratio of a drainage basin, several methods
can be used that depend on the choice of a dominant basin length
(e.g. the longest channel length and length along the main valley
axis to the drainage divide), basin area and basin orientation, and
produce non-unique results. The method of computing the conver-
gence angle of river basins as defined in Castelltort et al. (2009)
leads to a unique measure of drainage basin shape independent of
such choices. Considering a simplified rectangular basin of length \(L_b\)
and width \(W_b\), with \(L_b > W_b\), the convergence angle is defined as
the angle \(\alpha\) that determines the aspect ratio of a basin:

\[
\tan \alpha = \frac{1}{2} \frac{W_b}{L_b}.
\]

Instead of enclosing a natural basin within a synthetic rectangle of
dimensions \(L_b\) and \(W_b\), the convergence angle method is based on calcu-
lating source-outlet vectors (Fig. 1). A source-outlet vector is defined as
the vector between the current pixel and the basin outlet. After
extracting the river network in order to define the catchment boundary,
every pixel in the basin is linked with the outlet pixel in order to define
the population of source-outlet vectors. The convergence angle is such
that half of the source-outlet vectors possesses an azimuth comprised
between \(+\alpha\) and \(-\alpha\) of the main median basin azimuth, and is thus
found by taking half of the angular difference between the first
and third quartiles of the azimuths of all source-outlet vectors (Castellott
et al., 2009). Large or small convergence angles correspond to relatively wide or narrow basins respectively.

Using the SRTM1 (~30 m horizontal resolution) digital topography for North America and the SRTM3 (~90 m) topography for South America, Castelltort et al. (2009) studied basins ranging in size from 8 to 12,500 km². They assumed that the considered basins were close to their initial state of evolution, i.e. were little dissected as witnessed by the presence of flat-top interfluves and the limited local relief of 3 to 368 m with an average of 80 m. The regional slope $S_R$ of the surfaces on which these basins were formed ranges from 0.0085° to 6.98°. The obtained convergence angles are plotted in Fig. 2 and labeled by the region of provenance. These data show that basins having surface slopes of 1.5° generally display convergence angles of less than 8°, whereas basins on slopes of less than 1° display a large variety of convergence angles from ~5° to ~24°. Surface slope thus exerted an important control on the shape of river basins at their initiation. This result corroborates the initial assumptions of Zernitz (1932) on the influence of surface slope on catchment shape and the subsequent experimental results of Parker (1977) and Phillips and Schumm (1987).

This relationship between regional slope and basin shape results from the interaction between regional and local slopes ($S_R$ and $S_l$). The analytical relation proposed in Castelltort and Simpson (2006a) describes $\alpha$ of a rectangular drainage basin as the characteristic deviation of water flowing orthogonally to topographic contour lines as a result of superimposed local roughness and regional slope:

$$\alpha = \frac{1}{2} \arctan \phi,$$

where $\phi$ is the relative surface roughness corresponding to the ratio $S_l/S_R$, reflecting a simple surface roughness of wavelength $\lambda$, and amplitude $A_l (S_l = 2A_l/\lambda)$ on a tilted surface (see Fig. 3).

3. Numerical experiments

The aim of this study is to test the influence of slope and roughness on the shape of incipient river basins in a numerical model of landscape evolution. To reach that goal, we use the widely distributed CASCADE code (Braun and Sambridge, 1997; Braun and Yamato, 2010) because it computes water paths according to a generic steepest descent algorithm similar to that used in most other landscape evolution models.

3.1. Model setup

In our study, the model box is constituted by an inclined square surface of 100 x 100 km (Fig. 3) with $S_R$ being defined by the ratio between maximum elevation at the top side ($h$) and the side length of the model ($l$):

$$S_R = \arctan \left( \frac{h}{l} \right).$$

The resolution of the model ($r$) is 1 km, i.e., 101 nodes both in the $x$ and $y$ directions.

The initial meshing is irregular, with 250 m of noisy displacement in both $x$ and $y$ directions.

The value of $h$ changed according to the desired $S_R$ value imposed to the experiment. The surface roughness is simulated by a simple uniform white noise topography of amplitude $A$, equivalent to $S_R$, applied at the beginning of the experiments (Fig. 3). Using a Gaussian white noise rather than uniform noise does not significantly modify the results. Although it is beyond the scope of this study, a more complete investigation of the role of different spectra of noise on $\alpha$ of river basins would be valuable. $S_l$ is thus defined as:

$$S_l = \arctan \left( \frac{R}{\Omega} \right).$$

where $r$ corresponds to the resolution shown in Fig. 3. $S_l$ and $S_R$ are then systematically varied to study their respective influence on $\alpha$.

In the CASCADE, both fluvial and hillslope processes are taken into account. For fluvial processes, erosion is simulated by the difference between the river channels carrying capacity $q_c$ and the sediment load $q_s$ scaled on the bedrock erodibility (bedrock erodibility length scale $l_{er}$) and the river channel width ($w_c$) such as:

$$\frac{\partial h}{\partial t} = q_s - q_c,$$

where $\frac{\partial h}{\partial t}$ corresponds to the variation of topography through time due to fluvial erosion. $q_s$ is obtained by integrating the volume of rocks eroded from the upstream area and $q_c$ corresponds to the volume that can be carried by water per unit time. $q_s$ is thus obtained by the stream-power-law proportional to the local river slope $S$ and the drainage area $A$ (Kooi and Beaumont, 1994):

$$q_c = K_F S^n A^m,$$

where $K_F$ is the fluvial erosion/transport coefficient (in m yr⁻¹) varying essentially with precipitation rate, and $n$ and $m$ are power-law exponents. Tests of the influence of varying the m/n ratio are presented in Section 3.2. If $q_s$ is smaller than $q_c$, erosion takes place. At the opposite, when $q_s$ exceeds $q_c$, deposition takes place at a rate of:

$$\frac{\partial h}{\partial t} = \frac{q_c - q_s}{\Omega},$$

where $\Omega$ is the surface area linked to each integration point and defined by the spatial discretization.
For the hillslope processes, a diffusion equation is used to simulate the mass transport \( q \) linearly proportional to the river slope (Kooi and Beaumont, 1994; Braun et al., 2001)

\[
q = -K_d \cdot S
\]

where \( K_d \) is a diffusivity coefficient (in \( \text{m}^2 \text{yr}^{-1} \)) corresponding to the efficiency of the hillslope transport processes.

### 3.2. Influence of model parameters

The values of the different parameters used are shown in Table 1. We here present their influences and discuss our choices of parameters for the study of the effect of slope and roughness.

#### Table 1

Numerical model constants used in our experiments. *Parameter tested that demonstrated no significant influence on the value of the convergence angle \( \alpha \) (see text and Figs. 4 and 6 for details).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model width</td>
<td>( L )</td>
<td>100 km</td>
</tr>
<tr>
<td>time step</td>
<td>( \Delta t )</td>
<td>20 yr</td>
</tr>
<tr>
<td>Hillslope diffusivity</td>
<td>( K_d )</td>
<td>0 ( \text{m}^2 \text{yr}^{-1} )</td>
</tr>
<tr>
<td>Fluvial erosion/transport coefficient</td>
<td>( K_f )</td>
<td>5 \times 10^{-6} ( \text{m} \text{yr}^{-1} )</td>
</tr>
<tr>
<td>Uplift rate</td>
<td>( V_z )</td>
<td>5 \times 10^{-3} ( \text{m} \text{yr}^{-1} )</td>
</tr>
<tr>
<td>Bedrock erodibility length scale</td>
<td>( l_{\text{BR}} )</td>
<td>1 m</td>
</tr>
<tr>
<td>Channel width length scale</td>
<td>( W_c )</td>
<td>1 m</td>
</tr>
</tbody>
</table>

Fig. 3. Model setup. (a) Initial morphology of simulations corresponding to a simple tilted plane characterized by its regional slope \( S_R \) and the local slope \( S_l \) reflecting local roughness. (b) Topography after 500 ky in one experiment. Catchments are extracted and the convergence angle \( \alpha \) is computed for each catchment.

Fig. 4. Influence of the erosional parameters on the drainage basin convergence angle \( \alpha \). (a) Effect of diffusion \( (K_d) \). \( \alpha \) is computed for a roughness \( R \) of 10 m for five different regional slopes \( (0.0001^\circ, 0.001^\circ, 1^\circ, 2^\circ \text{ and } 3^\circ) \). (b) Effect of fluvial coefficient \( (K_f) \). As in (a), \( \alpha \) is computed for a roughness \( R \) of 10 m for five different regional slopes \( (0.0001^\circ, 0.001^\circ, 1^\circ, 2^\circ \text{ and } 3^\circ) \).
Diffusion leads to a “smoothing” of surface topography which does not produce changes in the final morphology of the river network. Tests presented in Fig. 4a show differences of 1° to 2° in $\alpha$ when $K_d$ is varied by four orders of magnitude over a very small regional slope. When the regional slope is increased to 1°, 2° and 3°, $K_d$ has no influence on the convergence angle. We thus neglect diffusion in other experiments ($K_d = 0$). Note that we have not tested the effect of using a non-linear slope-dependent transport law on hillslopes (e.g., Roering et al., 1999). Such a transport mode is essential in areas with high uplift and/or landslide-dominated incision. Our results are not relevant to such landscapes but rather to the initial stages of incision in areas undergoing topographic growth.

The effect of $K_f$ was tested by varying it by factors of 2 and 10 (Fig. 4b). Because it is a constant of proportionality, varying $K_f$ influences the rate of landscape evolution; given the other parameters used, if $K_f$ is increased by more than 10, the landscape erodes instantaneously. However, the obtained $\alpha$ values for the same regional slope differ only by 4° or less (Fig. 4b). We thus decided to keep a $K_f$ set to $5 \times 10^{-6}$ m yr$^{-1}$ throughout our study.

It is important to emphasize that with no diffusion and a constant coefficient of fluvial erosion, our experiments are designed for studying the influence of slope and roughness at the scale of dendritic convergent networks larger than incipient first-order streams. Indeed, the spacing of first-order channels has been demonstrated to be essentially controlled by the competition between advective and diffusive processes both under transport-limited (e.g., Simpson and Schlunegger, 2003) or detachment-limited (Perron et al., 2008) conditions.

**Fig. 5.** Influence of bedrock erodibility (erodibility length scale $l_{BR}$, left column) and channel width ($w_c$, right column). The parameter values are $K_f = 5 \times 10^{-6}$ m yr$^{-1}$, $R = 10$ m and $S_R = 0.01°$. Results demonstrate that these two parameters act in the same way and do not affect the convergence angle $\alpha$. 
Because $l_{Br}$ and $w_c$ both come as proportionality factors into the erosion law used in this study (Eq. (5)), we have performed tests to confirm that they have similar influence on the simulations (Fig. 5). Results show that these parameters also affect the rate of landscape incision but modify $\alpha$ only by less than 5°. We thus set both $l_{Br}$ and $w_c$ to an intermediate value of 1.

In order to study the influence of $m$ and $n$ on $\alpha$, we performed experiments using typical values of $m/n$, i.e. varying from 0.3 to 0.7 (Whipple and Tucker, 1999). Results are presented in Fig. 6 and show that varying the $m/n$ ratio influences only the rate of erosion of the landscape but modifies $\alpha$ only within a range of 9.78° to 12.68°, i.e. not significantly. In the experiments focusing on the influence of slope and roughness on $\alpha$, we have thus set $m/n$ to a conservative value of 0.5 with $n = 1$, as commonly used in the literature (e.g., Whipple and Tucker, 1999).

It is important to highlight that, since the numerical parameters such as $K_f$, $K_d$, $l_{Br}$, $w_c$, $m$ and $n$ are not well constrained from natural data, the insight gained from the experiments emanates more from the qualitative differences of network shape obtained with different local and regional slopes rather than from the absolute values.

3.3. Simulations results

Fig. 7 displays the fluvial landscapes obtained after 25,000 time steps (equivalent to 500 ka) for different values of local roughness ($0.1$, $1$, $10$, $100$ and $1000$ m) and regional slopes ($0.0001°$, $0.01°$, $1°$, $3°$ and $5°$). $\alpha$ is computed according to the method outlined above, for all basins with areas of 20 km² at least, and we then display the average $\alpha$ value for each experiment. Supplementary tests have been done for different values of minimum catchments and present almost the same results. In all experiments the water path over the surface is established quasi-instantaneously and the erosion wave that then propagates upstream tends to follow this pre-existing path. The role of dissection is limited to emphasizing the catchment form established early by water routing over the rough topography. The erosion processes are thus of limited importance in setting the landscape geometry, as also highlighted by the negligible impact of erosion parameters on $\alpha$ as explained in Section 3.2. In the experiments of Fig. 7, we have also investigated the evolution of $\alpha$ with time and the effect of the numerical grid resolution. Observation of drainage network evolution with time shows that reorganization is minimal and limited to nodes close to the divides in first-order basins. This is expressed by the evolution of $\alpha$ within a limited range of 1° to 3° for each experiment shown in Fig. 8. The spatial resolution of the numerical grid also does not affect $\alpha$ considerably (Fig. 9).

Therefore, in these experiments, the combination of initial roughness and regional slope determines the final basin aspect ratio ($\alpha$) almost entirely.

The obtained fluvial landscapes display dendritic networks that are visually different as a function of $\Phi$. As intuitively expected and analytically predicted (Castelltort and Simpson, 2006b), the fluvial networks are more straight and narrow when $S_R$ increases relative to $S_\Phi$.
to $S_l$ (top left of Fig. 7), and are more dendritic and wide when local roughness dominates over $S_R$ (bottom right of Fig. 7). For a constant slope (each row of Fig. 7), $\alpha$ of simulated networks increases with increasing roughness (from left to right). Similarly, for a constant roughness (each column of Fig. 7), $\alpha$ decreases with increasing $S_R$ (from bottom to top).

Fig. 10a shows the comparison of $\alpha$ for simulated drainage networks and natural ones as a function of $S_R$. The values obtained in the numerical experiments are similar to those of drainage networks established on surfaces found in nature. For constant roughness (each colored dotted line represents a constant value of roughness), $\alpha$ decreases with increasing $S_R$ as analytically predicted. The natural data (white dots and best fit black curve) are best matched by numerical experiments with local roughness of 100 m, i.e. $S_l$ of 2.8624° (Fig. 10b). Although local roughness of less than 100 m produces very small $\alpha$ values on large $S_R$ compared to natural data (Fig. 10b), it fits some of the lower bound on natural data (Fig. 10a). Local roughness of more than 100 m yields $\alpha$ values that are too large compared to natural values for networks on steep pristine slopes (Fig. 10b).

4. Discussion

The experiments conducted in this study show that the shape of catchments is extremely sensitive to the initial conditions of surface...
slope and roughness. The routing of water flow down the steepest slope in the CASCADE numerical model produces results that correspond well to analytical predictions and fit well to the observation of incipient drainage basin formation on large-scale tilted surfaces. In this case, the physical processes of erosion play only a minor role in structuring the drainage basin geometry which is rather controlled by the initial conditions of surface slope and roughness. It is thus possible to set up the initial roughness and slope of numerical experiments in order to obtain the desired catchment geometry needed for specific case studies. This can be particularly useful in studies that seek to invert a landscape’s history, by choosing the appropriate initial conditions.

Basin length-to-width ratios in nature vary over a wide range of values around central values of 2 to 3 (Montgomery and Dietrich, 1992; Hovius, 1996), i.e. $\alpha$ between 9° and 14°. To produce fluvial landscapes with such a classical natural geometry on a simple tilted plane, it is clear from our experiments that the initial slope has to be significantly lower than 1° and be characterized by a "reasonable" amplitude of local roughness less than 100 m over 2 km, i.e. less than 3° (Fig. 10). The experiments never produce basins with large $\alpha$ values (wide basins) as sometimes observed in nature. Such wide natural basins may result from structural control, tectonic or lithological, active or inherited, acting as an oriented roughness. This is the case for example at the front of the Himalayas, where large-scale orogeny-parallel anticlines divert the incipient fluvial systems and produce anomalously wide drainage basins (Hovius, 1996). Such a structural control can be input in numerical experiments in order to obtain wider basins. Beyond structural controls, however, one could introduce a different type of random noise on initial experiments in order to produce larger basins. In particular one could test the influence of having a spectrum of noise, i.e. different amplitudes at different wavelengths. However, a major drawback to constraining the input roughness comes from the absence of data on the roughness of natural surfaces at large scale in their pre-incision configuration. A third possible reason of never obtaining large $\alpha$ values in our experiments could be due to the water flowing algorithm which may force the water too straight down the regional slope. While the irregular mesh used in the CASCADE breaks the symmetry found on regular
and therefore introduces a variety of flow directions, this may not be sufficiently close to the natural flow variation found in nature on very low slopes. Introducing some stochasticity in the flow directions like in other landscape evolution models (Murray and Paola, 1994) or using different flow algorithms such as the D-Infinity fluvial landscape evolution models (Pelletier, 2004, 2010) could be explored.

A major remaining research problem is the non-uniqueness of the results: while the initial conditions seem to be appropriate, this is not sufficient to exclude the importance of erosion processes in nature which may lead to the same results. This outlines the need for future research in producing landscape evolution models considering more internal dynamics.

5. Conclusions

In this study we have explored the influence of initial conditions of slope and roughness on the shape of drainage basins in numerical experiments of simple tilted surfaces using the CASCADE code of landscape evolution. We show that simple rules of routing water downstream according to the steepest path are sufficient to account for a natural range of the incipient drainage basin aspect ratio, independently of the erosion processes at work. To produce numerical basins that respect the main aspect ratio of natural drainage basins, one must use very low slopes of less than 1° and local roughness slope of less than −3°. Numerical studies addressing real study cases may take advantage of the relation between local roughness and regional slope in order to produce catchment aspect ratios similar to the natural studied cases. Our simulations suggest that the maximum local slope that seems to account well for the natural observation is 10° (−3°). This value is a potential upper bound on the roughness of un-dissected surfaces in nature, for the first time obtained from a numerical model of landscape evolution as far as we know, within the limits of the model used. Accordingly, our results suggest that natural basins presenting anomalously large convergence angle (large width-to-length ratio) may either be controlled by anomalous roughness (structural or lithological), or have been strained tectonically. This study highlights the need for future research on natural observations of fluvial network changes and deformations, and on numerical implementation of better algorithms to reproduce river captures and drainage network reorganization that could also lead to wider drainage basins.

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